EFFECTIVE FIELD METHOD IN IDEAL PLASTICITY THEORY FOR COMPOSITE MATERIALS

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In studying usefully important problems of flow for multicomponent mixtures for deformation of solid materials prepared by the method of powder metallurgy the requirement arises of predicting limiting loads for these systems. It is assumed that a composite material consists of a uniform matrix and a random collection of ellipsoidal inclusions. Calculation of the effective plasticity limit for the material and a parameter describing its bulk compressibility is accomplished by means of a variant of the effective field method suggested in [1, 2]. The method is based on the problem of solving binary interaction of inclusions in an effective field assuming stress uniformity within each inclusion. An assumption has been used for uniformity of the dissipative function within inclusions and the matrix.

1. In a macrovolume with characteristic function W we consider a mixture of rigidly plastic components whose properties are described by a flow surface taking account of bulk compressibility:

$$I_2 + b(x) I_1^2 = k^2(x), \tag{1.1}$$

where $I_1 = \sigma_{ii}$ is first invariant of stress tensor σ_{ij}; $I_2 \equiv s_{ij}s_{ij}$ is second invariant of stress deviator $s_{ij} = \sigma_{ij} - \sigma_{\ell\ell}\delta_{ij}/3$. The matrix with plasticity limit $k(x) = k_0$ and parameter $b(x) = b_0$ characterizing its bulk compressibility contains a Poisson set $X = (V_k, x_k, \omega_k)$ $(k = 1, 2, \ldots)$ of ellipsoids v_k with characteristic function V_k, centers x_k, semiaxes $_k{}^i$ $(a_k{}^1 > a_k{}^2 > a_k{}^3)$, a set of Euler angles ω_k, and parameters k_k and b_k. There is ideal adhesion between components so that the field of displacement velocities $u_i(x)$ is continuous.

For (1.1) a local association rule for flow of components is observed [3-6]

$$\sigma_{ij} = k(x) \frac{\varepsilon_{ij} - \delta_{ij} d(x) \varepsilon_{ll}(x)}{\sqrt{\varepsilon_{mn}(x)\varepsilon_{mn}(x) - d(x)\varepsilon_{ll}^2(x)}} \tag{1.2}$$

[ε_{ij} is strain rate tensor, $d(x) = (3b(x) - 1)/9b(x)$]. By multiplying (1.2) in a scalar way by ε_{ij} we obtain a relationship for dissipative function $D(x) = \sigma_{ij}(x)\varepsilon_{ij}(x)$, by means of which the expression for the flow rule is simplified:

$$\sigma_{ij}(x) = \frac{k^2(x)}{D(x)}\varepsilon_{ij} - \frac{k^2(x)\,d(x)}{D(x)}\varepsilon_{ll}\delta_{ij}. \tag{1.3}$$

Similarly [5], we shall assume that the dissipative function is uniform within the limits of each component $\alpha = 0, 1, \ldots, N$, $D_\alpha = \text{const}$, and $D_\alpha = \sqrt{\langle k^2\varepsilon_{mn}\varepsilon_{mn} - k^2 d\varepsilon_{\ell\ell}{}^2\rangle_\alpha}$; here and below the following notations are adopted: $\langle(\cdot)\rangle_\alpha = \bar{V}_\alpha^{-1}\int(\cdot)V_\alpha(x)dx$, $\langle(\cdot)\rangle = \bar{W}^{-1}\int(\cdot)W(x)dx$ and $\langle(\cdot)|x_2; x_1\rangle$ is the nominal average for an assembly of an ergodic static uniform field $X(\cdot|x_1)$ assuming that at points x_1 and x_2 inclusions are found and $x_1 \neq x_2$; $\bar{V}_\alpha = \operatorname{mes} V_\alpha$; $V_0 = W\setminus V \equiv W\setminus \bigcup_{k=1} V_k$. Inclusions relate to different phases if they have different (even only one) parameters a_k, ω_k, k_k, b_k. By substituting (1.3) in equilibrium equation $\sigma_{ij,j} = 0$, we obtain

$$\nabla L_0 \nabla u = -V\nabla(L_1 - L_0)\nabla u, \tag{1.4}$$

where ∇ is operation of a symmetrical gradient, $\varepsilon \equiv \nabla u$, and isotropic tensors L_0 and L_1 with $x \in V_\alpha$ may be written in the form

$$L_0 \equiv (3k^0, 2\mu^0) \equiv 3k^0 N_1 + 2\mu^0 N_2, \quad L_1(x) = (3k_1^1(x), 2\mu^1(x)), \tag{1.5}$$
$$N_1 = \delta_{ij}\delta_{kl}/3,\ N_2 = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - (2/3)\delta_{ij}\delta_{kl})/2,$$

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$$3k^0 = k_0^2(1-3d_0),\quad 2\mu^0 = k_0^2,\quad 3k^1(x) = k_1^2(x)(1-3d_1)D_0/D_\alpha,$$
$$2\mu^1(x) = k_1^2 D_0/D_\alpha \quad (x \in V_\alpha).$$

Equation (1.4) with a specified accuracy conforms with similar relationships in the linearly elastic mechanics problem for composites [1, 2]. Since nonlinearity effects only develop in the dependence of L_1 on D_α, and L_0 = const, then problem (1.4) is a special case of an elastic physically nonlinear problem [7] and it may be solved as suggested in [1, 2] with a variant of the effective field method. In fact, Eq. (1.4) is reduced to an integral equation [1, 2] by means of fundamental solution [8]:

$$U_{ij}(x) = (8\mu^0\pi)^{-1}(\delta_{ij}|x|,_{pp} - \varkappa^0|x|,_{ij}),\ \varkappa^0 = (3k^0+\mu^0)/(3k^0+4\mu^0),$$
$$\varepsilon(x) = \langle\varepsilon\rangle + \int G(x-y)\{[L]\,\varepsilon(y) - \langle[L]\,\varepsilon\rangle\}\,dy$$

($G(x) = \nabla\nabla U(x)$, $[L^{(k)}] = (L_1(x_k) - L_0)V_k$ is a piecewise-constant tensor differing from zero only within inclusions, $[L] = \sum\limits_k([L^{(k)}])$; integration is carried out everywhere with respect to W.

We prescribe the structure of the composite by means of binary correlation function $\varphi(x_k, \omega_k|x_i, \omega_i)$, i.e., probable location of the k-th inclusion in assembly X with fixed i-th inclusion. We assume that φ is centrally symmetrical:

$$\varphi(x_k, \omega_k|x_i, \omega_i) = \psi(\omega_k)f_k(|x_i - x_k|)\overline{W}^{-1}. \tag{1.6}$$

Here $f_k(|x_i - x_k|) = 0$ with $x_k \in v_i'$ and $f_k(|x_i - x_k|) = n_k$ with $x_k \notin v_i'$; v_i' is sphere radius $a_{ik} = \max\limits_j(a_i^j + a_k^j)$ with a center at x_i; n_k is calculated concentration of inclusions of component $X_k \subset X$, i.e., connected with their volume concentration $c_k = (4/3)\pi a_k{}^1 a_k{}^2 a_k{}^3 n_k$, $c = \sum\limits_{k=1}^{N} c_k$; $\psi(\omega_k)$ is orientation density distribution for inclusions of component X_k.

We fix the inclusion with number i, then by separating from the right-hand part of (1.5) the term equal to the local external field $\bar{\varepsilon}_i$ in which the i-th inclusion is found, by means of averaging with respect to φ (1.6), we find

$$\langle\bar{\varepsilon}_i\rangle = \langle\varepsilon\rangle + \int G(x-y)\{\langle[L(y)]\,\varepsilon(y)\,V(y;\,x_i)\,|\,y;x_i\rangle - \langle[L]\,\varepsilon\rangle\}dy \tag{1.7}$$

$\left((V(y;\,x) = \bigcup\limits_{k=1} V_k \setminus V_i(x_i)\right)$. In order to close (1.7) it is necessary to establish the relationship between $\overline{\varepsilon_i}$ and $\varepsilon(x_i)$, and also between $\bar{\varepsilon}(x_i)$ and $\bar{\varepsilon}(y)$.

2. We solve problem (1.7) for the case when assembly X consists of one and two inclusions. Since according to the assumption L_0, L_1 = const in the matrix and inclusions, then Eq. (1.7) with a specified accuracy conforms with the similar relationship of linear elasticity theory for composites [1] and for a single inclusion in the prescribed uniform field at infinity $\varepsilon^0 \equiv L_0^{-1}\sigma$, the fields for strain rate ε_i and stresses σ_i within an inclusion are also uniform:

$$\varepsilon_i(x) = A_i\varepsilon^0,\ A_i = (I + P_i[L])^{-1},\ \sigma_i = B_i\sigma \equiv L_1A_iL_0^{-1}\sigma \tag{2.1}$$

($x \in v_i$, and constant tensor $P_i \equiv -\int G(x-y)V_i(y)dy$ is known [8]).

Solution of the problem of binary interaction of inclusions in an infinite matrix with prescribed uniform field at infinity ε^0 may be constructed by the method of successive approximations. From the equation

$$\varepsilon(x) = \varepsilon^0 + \int G(x-y)\,[L]\,\varepsilon(y)\,[V_i(y) + V_j(y)]\,dy \tag{2.2}$$

taking account of the first iterations we find the relationship required with solution (1.7)

$$G(x_i - x_j)\,[L^{(j)}]\,\varepsilon(x_j)\bar{V}_j = G(x_i - x_j)R_j(\varepsilon^0(x_j) + G(x_i - x_j)R_i\varepsilon^0(x_i)), \tag{2.3}$$

in obtaining which an assumption is made about the point nature of inclusions [1, 2] and the uniformity of a field $\varepsilon(x_i)$ within each inclusion; $R_i = [L^{(i)}]A_i\tilde{V}_i$.

It is noted that with these assumptions problem (2.2) may be solved by linear algebra methods:

$$G(x_i - x_j)R_j\bar{\varepsilon}(x_j)V_j = G(x_i - x_j)R_j\{(Z^{-1})_{ji}\varepsilon^0(x_i) + (Z^{-1})_{jj}\varepsilon^0(x_j)\}, \tag{2.4}$$

where $(Z^{-1})_{ij}$ is a matrix inverse to matrix Z with elements Z_{mn} (m, n = 1, 2) in the form of a submatrix

$$Z_{mn} = I\delta_{mn} + G(x_m - x_n)R_n(1 - \delta_{mn}). \tag{2.5}$$

In (2.4) and (2.5) $\bar{\varepsilon}(x)$ and $\varepsilon_0(x)$ vectors of dimension 6 are presented and [L], A, and R by matrices 6 × 6; the rule for converting from a tensor of the second and fourth ranks to vectors and matrices is described in [8]. Thus, inclusion v_i is in a uniform field depending on geometric and mechanical properties of the inclusion in question.

3. In obtaining relationships for $\langle\bar{\varepsilon}_i\rangle$ from (1.7) we use hypotheses for an effective field [1, 2] according to which field $\bar{\varepsilon}_i$ is uniform in the vicinity of each point inclusion and it depends on the properties of this inclusion; each pair of inclusions v_i, v_j is in its own effective field $\bar{\varepsilon}_{ij}$ independent of the properties of the pair being studied. These hypotheses make it possible to transform (1.7), (2.1), and (2.4) to a set of equations in relation $\langle\bar{\varepsilon}_i\rangle$ by replacing $\varepsilon^0(x_i)$ in (2.1) and (2.4) by $\langle\bar{\varepsilon}_i\rangle$:

$$\langle\bar{\varepsilon}_i\rangle = \langle\varepsilon\rangle + \sum_{\nu=1}^{N} P(V_{i\nu}^0)R_\nu n_\nu\langle\bar{\varepsilon}_\nu\rangle + \sum_{\nu=1}^{N}\int G(x_i - x)R\{(Z^{-1})_{\nu\nu} - I\}\times \\ \times(1 - V_{i\nu}^0)n_\nu\,dx\,\langle\bar{\varepsilon}_\nu\rangle + \sum_{\nu=1}^{N}\int G(x_i - x)R_\nu(Z^{-1})_{\nu i}n_\nu(1 - V_{i\nu}^0)\,dx\,\langle\bar{\varepsilon}_i\rangle. \tag{3.1}$$

Integrals in (3.1) converge absolutely since $|x_i - x| \to \infty$, $(Z^{-1})_{\nu\nu} \to I$, and $(Z^{-1})_{\nu i} \to 0$. Tensors of the second and fourth rank in (3.1) are presented in the form of vectors of dimension (6 × 1) and matrix (6 × 6). We form from vector $\langle\varepsilon\rangle^T$ a vector of dimension (6 × 1) $\langle E\rangle^T = (\langle\varepsilon\rangle^T, \ldots, \langle\varepsilon\rangle^T)$ (T is the sign of transposition), and similarly we form $\langle\bar{E}\rangle^T = (\langle\bar{\varepsilon}_1\rangle^T, \ldots, \langle\bar{\varepsilon}_N\rangle^T)$. Then system (3.1) may be presented in matrix form

$$Y\langle\bar{E}\rangle = \langle E\rangle, \tag{3.2}$$

where elements of the matrix $Z = Z_{ij}$ (i, j = 1, ..., N) serve submatrices

$$Y_{ij} = \delta_{ij}\left(I - \sum_{\nu=1}^{N} G(x_i - x)R_\nu(Z^{-1})_{\nu i}n_\nu(1 - V_{i\nu}^0)\,dx\right) - \\ - P(V_{ij}^0)R_j n_j - \int G(x_i - x)R_j\{(Z^{-1})_{jj} - I\}n_j(1 - V_{ij}^0)\,dx.$$

Turning to matrix Y we find that

$$\langle\bar{\varepsilon}_i\rangle = \sum_{j=1}^{N}(Y^{-1})_{ij}\langle\varepsilon\rangle. \tag{3.3}$$

In order to obtain an analytical expression for $\langle\bar{\varepsilon}_i\rangle$ we use, together with accurate solution (2.4), iteration approximation (2.3); then $(Z^{-1})_{\nu\nu} = I$, $(Z^{-1})_{\nu i} = G(x_i - x)R_\nu$. This leads to the situation that part of integrals in (3.1) disappear and the rest may be calculated analytically. In addition, if we adopt the assumption that $\langle\bar{\varepsilon}_i\rangle = \langle\varepsilon\rangle$ = const [1, 2] and $V_{1\nu}^0 = V_1^0$ is a sphere of radius $2a_i^1$, then by averaging (3.1) with respect to ω_i and a_i we obtain

$$\langle\bar{\varepsilon}\rangle = D^\varepsilon\langle\varepsilon\rangle,\ D^\varepsilon = \left(I - P(V_i^0)\sum_\nu\langle R_\nu n_\nu\rangle - \sum_\nu\int J^0(x)\,dx\right)^{-1}. \tag{3.4}$$

Here $J^0(x) = \langle\!\langle G(x_i - x)R_\nu G(x_i - x)R(1 - V_i^0)\rangle\!\rangle_{i\nu}$; $\langle\!\langle\cdot\rangle\!\rangle_{i\nu}$ means averaging with respect to ω_i, a_i, ω_ν, a_ν in a sphere of radius $|x_i - x|$ with center x_i. Similarly, we find the stress concentration tensor D^σ in inclusions of component i which in analytical form are

$$\langle\bar{\sigma}\rangle = D^{\sigma}\langle\sigma\rangle,\ D^{\sigma} = \left(I - Q(V_i^0)\sum_{\nu}\langle\overline{R}_\nu n_\nu\rangle - \sum_{\nu}\int J^0(x)\,dx\right)^{-1},$$
$$Q_\nu = L_0(I - P_\nu L_0),\ \overline{R}_\nu = B_\nu[M^{(\nu)}]\bar{V}_\nu,\ [M^{(\nu)}] = \{L_1^{-1}(x) - L_0^{-1}\}V_\nu. \tag{3.5}$$

Tensor $J^0(x)$ in the case when $\langle R_\nu\rangle$ ($\nu = 1, \ldots, N$) are isotropic, is also isotropic, and its values is given in [1]. Tensors D^ε, $D^{\varepsilon i} \equiv \sum_{j=1}^{N}(Y^{-1})_{ij}$ in (3.3) characterizing concentration of strain rates caused by surrounding inclusions makes it possible to determine effective characteristics of the material. For this we average local flow rules for representative volume W; in tensor form we obtain $D_0\langle\sigma\rangle = L_0\langle\varepsilon\rangle + \sum_i[L^{(i)}]\langle\varepsilon\rangle_i$. Taking account of the assumption for uniformity of field $\langle\bar{\varepsilon}_i\rangle$ and relationships (2.1), (3.3), and (3.4) we find that

$$D_0\langle\sigma\rangle = L^*\langle\varepsilon\rangle,\ L^* = L_0 + \sum_i\langle R_i n_i\rangle D^{\varepsilon i}; \tag{3.6}$$

$$(M^*\langle\sigma\rangle)\langle\sigma\rangle = D^*/D_0,\ D^* = \langle\sigma_{ij}\rangle\langle\varepsilon_{ij}\rangle \tag{3.7}$$

$[M^* = (L^*)^{-1}]$. If instead of relationship (3.3) we use (3.4), it is necessary to replace $D^{\varepsilon i}$ by D^ε in (3.6).

4. Until now it has been assumed that $L_1(x_k)$ is known, but according to the suggestion these tensors depend through dissipative functions D_0 and D_k on unknown fields for stresses and strain rates in components. In calculating D_0 and D_k we make a number of assumptions. If similar to [3], a hypothesis is taken about the absence of fluctuations D_α ($\alpha = 0, 1, \ldots, N$) not only in component X_α, but also in the whole of volume W, then (3.7) is an explicit expression for the flow macrosurface of a composite material. In fact, with $D_\alpha = D^* = \mathrm{const}$, $D_0/D_k = 1$ ($k = 1, \ldots, N$), and tensor L_1, and this also means R_k, D^ε, and M do not depend on fields for stresses and strain rates. In the general case L^* is anisotropic, but with an equally probable orientation of inclusion L^* is isotropic and the flow macrosurface for the material will be prescribed by an equation similar to (1.1) with effective parameters

$$k^* = (2L_2^*)^{1/2},\ b^* = 2L_2^*/(9L_1^*) \tag{4.1}$$

[$3L_1^*$ and $2L_2^*$ are isotropic components for tensor $L^* = (3L_1^*, 2L_2^*)$].

We weaken the assumption about uniformity of the local dissipation function and we shall assume that D_α ($\alpha = 0, 1, \ldots, N$) is only constant within the component X_α in question. Similar to [5, 6] we take an approximation for the dissipative function $D_\alpha = \sqrt{\langle\bar{L}^\alpha\varepsilon)\varepsilon\rangle_\alpha}$, where $\bar{L}^\alpha = L^{(\alpha)}D_\alpha D_0^{-1}$, $L^{(\alpha)} = L_0$ with $V_0(x) = 1$ and $L^{(\alpha)} = L_1(x_\alpha)$ with $x \in v_\alpha$. In view of the stress field uniformity within inclusions, we assume approximately that $D_k^2 = \langle(\bar{L}^k\varepsilon)\varepsilon\rangle_k = \langle\bar{L}^k\varepsilon\rangle_k\langle\varepsilon\rangle_k$ ($k = 1, \ldots, N$) [5, 6], and more accurate relationships may be obtained by means of estimating the correlation function stress fields in inclusions by the procedure in [2], and for D_0 from manifest relationships

$$\langle V_k'\varepsilon'\rangle = c_k(\langle\varepsilon\rangle_k - \langle\varepsilon\rangle),\ (1-c)\langle(L_0\varepsilon)\varepsilon\rangle_0 =$$
$$= \langle L_0\varepsilon\rangle\langle\varepsilon\rangle + \langle(L_0\varepsilon')\varepsilon'\rangle - \sum_k c_k\langle L_0\varepsilon\rangle_k\langle\varepsilon\rangle_k \tag{4.2}$$

(a prime denotes fluctuation $f' = f - \langle f\rangle$). Then considering that with transformation of the volumetric integral by the first Green equation, the value of surface integral with respect to ∂W referres to mes W tends toward zero with mes W $\to \infty$, and by using Eqs. (1.4) and (3.6) we obtain

$$\langle(L_0\varepsilon')\varepsilon'\rangle = -\sum_k c_k\langle[L^{(k)}]\varepsilon\rangle_k(\langle\varepsilon\rangle_k - \langle\varepsilon\rangle). \tag{4.3}$$

In deriving the last equality as in [5], only the first term in expansion (2.3) it is considered, and more accurate estimates may be found by the procedure in [2]. Thus, dissipative functions only depend on uniform fields $\langle\varepsilon\rangle$ and $\langle\varepsilon\rangle_k$, and this means that they may be expressed by means of relationships (2.1), (3.4), (4.2), and (4.3) in terms of $\langle\varepsilon\rangle$:

$$D_k^\varepsilon = (\bar{L}^k A_k D^\varepsilon\langle\varepsilon\rangle)(A_k D^\varepsilon\langle\varepsilon\rangle),$$
$$D_0^2 = (1-c)^{-1}\left\{(L^*\langle\varepsilon\rangle)\langle\varepsilon\rangle - \sum_k c_k(\langle L^{(k)}A_k D^\varepsilon\rangle\langle\varepsilon\rangle)(\langle A_k D^\varepsilon\rangle\langle\varepsilon\rangle)\right\}. \tag{4.4}$$

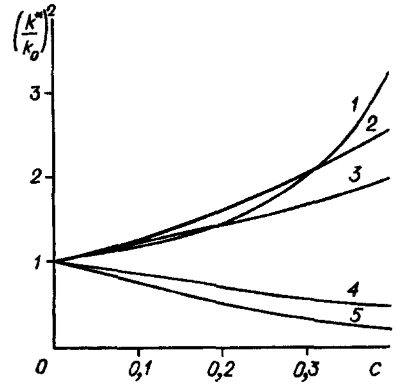

Fig. 1

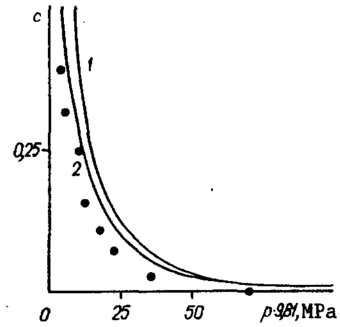

Fig. 2

Since in Eqs. (2.1) and (3.4) A_k and D^ε depend on dissipative functions D_0 and D_k, then in the general case D_0 and D_k may be obtained by the method of successive approximations. In fact, in the zero iteration $D_\alpha = D = \text{const}\ \forall\ \alpha$, then zero approximations A_k and D^ε do not depend on D_α and the first approximation for D_α may be found by Eq. (4.4); then by (1.5), (2.1), and (3.4) we calculate the first approximation of L_k, A_k, and D^ε, move to (4.4), etc.

5. We consider important cases in practice when it is possible to plot L^* in explicit form. Here we refer to materials with absolutely rigid inclusions and pores when $D_k = 0$ ($k = 1, \ldots, N$) and $L_1(x_k)$ do not depend on dissipation functions D_α ($\alpha = 0, 1, \ldots, N$). For equiprobable oriented inclusions, components of isotropic tensor $J_{ij} = (3J_{ij}{}^1, 2J_{ij}{}^2)$ are known [1]:

$$3J_{ij}^1 = 2(\widehat{3k_i})(\widehat{2\mu_j})\xi^2|r|^{-6},$$
$$2J_{ij}^2 = \frac{2}{5}\left[(\widehat{3k_j})(\widehat{2\mu_i})\xi^2 + (\widehat{2\mu_i})(\widehat{2\mu_j})(7\gamma^2 + 2\xi\eta - \eta^2/4)\right]|r|^{-6},$$
$$\xi = (3k^0 + 4\mu^0)^{-1},\ \eta = (3\mu^0)^{-1},\ \gamma = (3k^0 + \mu^0)(3\mu^0(3k^0 + 4\mu^0))^{-1},$$
$$(\widehat{3k_i},\ \widehat{2\mu_i}) = \prod_{n=1}^{3} a_i^n \int A_i\left[L^{(i)}\right] d\omega_i.$$

Let spherical pores of uniform size be placed in a plastically incompressible matrix. Then $L_0 = (\infty, k_0{}^2)$, $D^\sigma = ([1 - (29/24)c]^{-1}, [1 - (35/24)c]^{-1})$, $J(r) = (5/3, 11/3)a^6/|r|^{-6}$, and values of D^σ for plane spheroidal pores may be obtained similar to [9]. Tensors A and R do not depend on dissipation functions, which makes it possible to find

$$L^* = k_0^2(2/c - 29/12,\ [1 - (35/24)c][1 + (5/24)c]^{-1}),\ D^* = (1 - c)D_0$$

and effective parameters for the material

$$(k^*)^2 = k_0^2(1 - c)(1 - (35/24)c)(1 + (5/24)c)^{-1},$$
$$b^* = c(1 - (35/24)c)[6(1 + (5/24)c)(1 - (29/24)c)]^{-1}. \tag{5.1}$$

It is noted that similar estimates

$$(k^*)^2 = k_0^2(1 - c)(1 + (2/3)c)^{-1},\ b^* = c/6(1 + (2/3)c)^{-1}, \tag{5.2}$$

found using the hypothesis of "strong isotropy" [3] have a difference of the first order of smallness with respect to c from (5.1). Curves 4 and 5 in Fig. 1 were calculated by Eqs. (5.2) and (5.1), respectively. In Fig. 2 are experimental data [10] (points) for the density of sintered electrolytic nickel subjected to uniaxial compression in a mold ($k_0 = 1.14$ MPa). From the associated flow rule relating (1.1) with effective parameters (5.2) and (5.1), curves 1 and 2 in Fig. 2 are calculated for the dependence of pressure p within this mold on c asymptotically close with small c and differing by 30% for p with $c \sim 0.40$. More accurate curve 2 will shift to the left on axis p the more ellipsoidal inclusions differ from spheroidal inclusions adopted in calculations of (5.1).

We consider the opposite case when with quite low stresses $\langle\sigma\rangle$ inclusions behave as rigid particles. The latter start to deform on reaching critical stresses. For undeformed inclusions in a plastically incompressible matrix $k_1 = \infty$, $b_0 = 0$,

$$L^* = (0, k_0^2[1 + (5/2)c\{1 - (31/16)c\}^{-1}]);$$
$$(k^*)^2 = k_0^2(1 - c)[1 + (5/2)c\{1 - (31/16)c\}^{-1}]. \tag{5.3}$$

In Fig. 1, curves 1-3 were calculated by Eqs. (5.3) from [5] and [3], respectively. In cases (5.1) and (5.3) the relative change in effective parameters L^*/L_0 correlates with the relative change in effective elasticity moduli [1, 2] where also provided is a comparison of experimental data for relative viscosity of Newtonian suspensions with calculations by different methods. It is shown that use of the effective-field method suggested [1, 2] makes it possible with $c > 0.4$ to refine markedly (by a factor of two) the calculated shear modulus compared with other methods, which agrees with the experiment. The values of stress concentration tensor $D^\sigma = (1 + (9/16)c)^{-1}$ obtained make it possible to find the critical value of macrostresses $\langle\sigma\rangle$ at which stress inclusions reach the plasticity limit:

$$(25/4)(1 + (9/16)c)^{-1} I_2^* + d_1 I_1^{*2} = k_1^2.$$

After reaching a critical stress determined by invariants I_1^* and I_2^*, the rheological model is described by general relationships (3.4)-(3.6), taking account of inclusion plasticity.

Thus, the effective field method suggested makes it possible to take account of stress-field inhomogeneity in the matrix and binary interaction of inclusions, and it leads to refinement of calculated effective parameters of composite material plasticity in the example considered.

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